ApacheSim Calculation Methods

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1. Introduction

Apache Simulation is a dynamic thermal simulation program based on first-principles mathematical modelling of the heat transfer processes occurring within and around a building. ApacheSim qualifies as a Dynamic Model in the CIBSE system of model classification, and exceeds the requirements of such a model in many areas.

The program provides an environment for the detailed evaluation of building and system designs, allowing them to be optimised with regard to comfort criteria and energy use.

Amongst the issues that can be addressed with ApacheSim are:

- Thermal insulation (type and placement)
- Building dynamics & thermal mass
- Building configuration and orientation
- Climate
- Glazing properties
- Shading, solar gain & solar penetration
- Casual gains
- Air-tightness
- Natural ventilation
- Mechanical ventilation
- HVAC systems
- Mixed-mode systems

Within ApacheSim, conduction, convection and radiation heat transfer processes for each element of the building fabric are individually modelled and integrated with models of room heat gains, air exchanges and plant. The simulation is driven by real weather data and may cover any period from a day to a year. The time-evolution of the building's thermal conditions is traced at intervals as small as one minute.

Results output by the simulation include:

- Comfort statistics
- Energy consumption
- Carbon emissions
- Room load statistics
- Plant sizes
- Detailed performance measures including hourly room temperatures (air, mean radiant and dry resultant), humidities, plant loads, casual gains and air exchanges
- Surface temperatures for comfort studies or CFD boundary conditions

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2. Simulation Principles

ApacheSim deals separately with each of the fundamental heat transfer and control processes affecting building thermal performance. The methods used to model these processes are described in the following sections.

3. Heat Conduction and Storage

3.1. Heat Conduction and Storage Fundamentals

The time-evolution of the spatial temperature distribution in a solid without internal heat sources is governed by the following partial differential equations:

$$\underline{W} = -\lambda\nabla T \tag{1}$$

$$\nabla.\underline{W} = -\rho c\partial T / \partial t \tag{2}$$

where

$T(x,y,z,t)$ is the temperature (ºC) in the solid at position (x,y,z) and time t

$\underline{W}(x,y,z,t)$ is the heat flux vector (W/m^2) at position (x,y,z) and time t

λ is the conductivity of the solid (W/m^{2}K)

ρ is the density of the solid (kg/m^3)

c is the specific heat capacity of the solid (J/kgK)

Equation 1 and 2 are expressions of the principles of conduction heat transfer and heat storage, respectively.

The heat diffusion equation (in its most general form in which λ, ρ, and c may vary with position) then follows:

$$\nabla.(\lambda\nabla T) = \rho c\partial T / \partial t \tag{3}$$

It is also necessary to consider heat storage in air masses contained within the building. The model of this process is

$$Q = c_p\rho_a VdT_a / dt \tag{4}$$

where

Q is the net heat flow into the air mass (W)

c_p is the specific heat capacity of air at constant pressure (J/kgK)

ρ_a is the air density (kg/m^3)

V is the air volume (m^3)

T_a is the air temperature (ºC)

3.2. Modelling Assumptions

In ApacheSim, conduction in each building element (wall, roof, ceiling, etc) is assumed to be uni-dimensional. Furthermore, the thermo-physical properties λ, ρ, and c of each layer of the element are assumed to be uniform within the layer. Under these assumptions equation 0 may be written

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$$\partial^2 T / \partial x^2 = \frac{\rho c}{\lambda} \partial T / \partial t$$

(5)

The system of equations is closed by the application of appropriate boundary conditions and the stipulation that *W* is continuous at the layer boundaries.

3.3. Discretisation

ApacheSim adopts a finite difference approach to the solution of the heat diffusion equation. This involves first replacing the element with a finite number of discrete nodes at which the temperature will be calculated.

In the spatially-discretised representation, equation 5 takes the form

$$\frac{T_{n-1} - 2T_n + T_{n+1}}{\delta_n^2} = -(\rho c / \lambda) \partial T / \partial t$$

(6)

where

T_n is the temperature (ºC)at node *n* and

δ_n is the local node spacing (m).

Nodes are distributed within the layers in such a way as to ensure accurate modelling of the heat transfer and storage characteristics for the chosen time-step. This choice is based on constraints imposed on the Fourier number

$$F = (\lambda / \rho C) \Delta / \delta_n^2$$

(7)

where

Δ is the simulation time-step (s).

In this process each layer may be assigned many nodes.

Next, the time variable is discretised. A variety of schemes may be adopted for this stage.

Explicit methods use a forward-difference scheme, which uses present and future values of the nodal temperature to express the temperature time derivative $\partial T / \partial t$ at the present time:

$$\dot{T}_n^j = (T_n^{j+1} - T_n^j) / \Delta$$

(8)

where

T_n^j is the temperature (ºC) at node *n* and time-step *j*,

$\dot{T}_n^j$ is the time derivative of temperature (K/s) at node *n* and time-step *j*.

Pure-implicit methods use a backward-difference scheme, in which the computed time derivative is applied one time-step in the future:

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$$\dot{T}_n^{j+1} = (T_n^{j+1} - T_n^j)/\Delta$$

(9)

The time derivatives in these equations are equated with $\partial T/\partial t$ in equation to establish a model of heat conduction that is discretised in both space and time.

To improve accuracy and stability a combination of explicit and implicit time-stepping is often used. The Crank-Nicholson semi-implicit method is an example of such a scheme. Another is the 'hopscotch' method, which applies explicit and implicit time-stepping to alternate nodes of the construction. This is the method adopted by ApacheSim. The advantages of this method are a high level of accuracy combined with very efficient computation.

A full description of finite difference methods may be found in Myers [3].

3.4. Air Gaps

Air gaps in both opaque and transparent constructions are modelled as pure resistances:

$$W = (T_1 - T_2)/R$$

(10)

where

W is the heat flow across the air gap

T_1 and T_2 are the temperatures of the surfaces adjacent to the air gap

R is the combined radiative/convective resistance of the air gap

The value of R is set as a parameter of the air gap layer.

3.5. Boundary conditions

The boundary conditions for conductive elements of the building are dictated by conditions in the spaces either side of the element. These spaces may be internal or external. Where an adjacent room has been rendered inactive (for example by it being assigned in inactive layer, or not being ticked for inclusion in a <VE> Compliance analysis) the conditions on the far side are assumed to be identical to those on the near side (a *reflexive boundary condition*). In this way the thermal mass of the element is taken into account, while the time-averaged heat transfer through it tends to zero.

3.6. Air Mass and Furniture Modelling

The dynamics of heat storage in the room air masses is described by Equation 4, in which *Ta* is the bulk air temperature of the room (in the *stirred tank* representation) and *V* is the room volume.

At the user's option, the effect of heat storage in the furniture may be incorporated into the analysis. A facility is offered for modelling furniture on the assumption that its temperature closely follows that of the air. Under this

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assumption its effect is to increase the effective thermal mass of the air by a factor termed the *furniture mass factor (f_f)*. In this case, equation 4 becomes

$$Q = (1 + f_f) c_p \rho_a V dT_a / dt$$

(11)

If furniture is to be ignored, f_f should be set to 0.

In cases where the furniture has substantial thermal capacity, it is best to model it by introducing additional internal walls, with suitable thermal properties, into the room model.

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4. Convection Heat Transfer

4.1. Convection Fundamentals

Convection is the transfer of heat (and in general other physical quantities) resulting from the flow of fluid over a surface. For the purpose of the present discussion the fluid is air and the surface is an element of a building. If the convective air flow is driven by external forces – for example wind or mechanical ventilation – it is referred to as *forced convection*. The term *natural convection* describes convection arising from buoyancy.

It is found by experiment that convective heat transfer can be accurately described by equations of the form

$$W = K(T_a - T_s)^n \tag{12}$$

where

W is the heat flux (W/m^2) from the air to the surface,

T_a is the bulk air temperature (ºC),

T_s is the mean surface temperature (ºC), and

K and n are coefficients.

For forced convection at sufficiently high air velocities it is found that, to a good approximation,

$$n = 1 \tag{13}$$

and the process is thus *linear*.

For natural convection, although *n* is usually somewhat greater than 1, its value is often sufficiently close to 1 for the approximation

$$W = h_c(T_a - T_s) \tag{14}$$

to be reasonably accurate. In this relation,

h_c is the convective heat transfer coefficient.

To cater for significant departures of n from unity, two approaches are possible.

In the first approach, the heat transfer equation (12) is *linearised* into the form (14), using a constant value of *hc* that gives a good approximation to the true behaviour at typical values of the temperature difference.

The second approach re-introduces the nonlinearity by allowing *hc* to be a function of the temperature difference. The value of *hc* is updated successively with computed values of this difference and convergence to a consistent solution is achieved by iteration.

ApacheSim gives the user control over which of these approaches is adopted.

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4.2. Exterior Convection

Convection occurring at the external surfaces of the building is predominantly wind-driven forced convection.

In ApacheSim external forced convection is modelled with a wind speed dependent convective heat transfer coefficient calculated from McAdams' empirical equations [1]

$$h_c = 5.6 + 4.0v \qquad (v < 4.88)$$

(15)

$$h_c = 7.2v^{0.78} \qquad (v \geq 4.88)$$

(16)

where

v is the wind speed (m/s) read from the simulation weather file.

Variables on the simulation weather file are recorded at hourly intervals. Linear interpolation is applied between the recorded values to compute values at each simulation time-step.

Provision is made in the constructions database program APcdb for the user to override this calculation procedure with a fixed value for the external convection coefficient.

4.3. Interior Convection

The ApacheSim user has a number of options for modelling convection heat transfer between air masses inside the building and the adjacent building elements:

- Fixed convection coefficients specified by CIBSE
- Variable convection coefficients calculated according to CIBSE procedures
- Variable convection coefficients calculated from the relations proposed by Alamdari & Hammond.
- User-specified fixed convection coefficients

The first three options may be selected from the Simulation Options facility of the ApacheSim interface. The fourth option will apply to any constructions for which fixed internal surface coefficients are applied in the constructions database program APcdb. For such constructions the fixed value will override the method selected in the Simulation Options interface.

4.4. CIBSE Fixed Convection Coefficients

The CIBSE 'Simple Model' for Heat Loss and Heat Gain calculations [1] are based on a constant (average) convection coefficient for internal surfaces:

$$h_c = 3.0$$

(17)

This value is applied to all internal surfaces if the option 'CIBSE fixed values' is selected in Simulation Options.

4.5. CIBSE Variable Convection Coefficients

CIBSE Guide Volume C [2] provides a procedure for calculating convection coefficients as functions of surface orientation, air-surface temperature difference and mean room air velocity. Since these coefficients are dependent on air-surface temperature difference and other dynamically varying simulation variables they must be applied as part of an iterative calculation procedure.

The internal convection coefficient is expressed as

$$h_c = fC\Delta T^{n-1}$$

(18)

where

$$\Delta T = \left| T_a - T_s \right|,$$

C is a coefficient depending on surface orientation,

f is a coefficient depending on mean air speed and

n is an exponent (as in equation 0).

Values for C, n and f are taken from CIBSE Guide C Tables C3.12 and C3.13:

Effect of surface orientation and temperature difference

Surface type	C	n
Vertical surfaces	1.4	1.33
Horizontal surfaces (upward heat flow)	1.7	1.33
Horizontal surfaces (downward heat flow)	0.64	1.25

Effect of mean air velocity

Mean room air speed (m/s)	f
0.0	1.0
0.5	1.3
1.0	1.7
2.0	2.4
3.0	3.1

In ApacheSim values of f are calculated from the following formula, which reproduces the values in Table C3.13 to sufficient accuracy:

$$f = 1.0 + 0.7v$$

(19)

where

v is the mean room air velocity (m/s)

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Values of mean room air velocity are estimated in ApacheSim from ventilation rates and room geometry.

4.6. Alamdari & Hammond Convection Coefficients

Alamdari & Hammond [4] established by empirical means a procedure for calculating convection coefficients for internal surfaces. Since they vary with temperature difference, these coefficients must be applied within an iterative calculation procedure.

For vertical surfaces, and horizontal surfaces for which the convective heat flow is upward, the Alamdari and Hammond expression is

$$h_c = [\{a(\Delta T/L)^{1/4}\}^6 + \{b\,\Delta T^{1/3}\}^6]^{1/6} \tag{20}$$

where

$\Delta T = |\,T_a - T_s\,|$

L is the *characteristic length* of the surface, and

a and b are coefficients.

For a wall or window, the characteristic length, L, is the height of the space of which it forms a boundary. For a horizontal surface, L is the diameter of the space.

The coefficients a and b are set as follows:

	a	b
Vertical surface	1.50	1.23
Horizontal surface (upward heat flow)	1.40	1.63

For horizontal surfaces for which the convective heat flow is downward, the expression is

$$h_c = 0.6\,(\Delta T/L^2)^{1/5} \tag{21}$$

All values of h_c are subject to a minimum of 1.0 W/m^{2}K on the grounds that there will always be some air movement generated by convection at other surfaces or by ventilation.

4.7. User-specified Fixed Convection Coefficients

The constructions database program APcdb provides an option to fix the convection coefficients for any construction type. If this option is selected, the specified convection coefficient will be applied wherever the construction type is used in the building. If a fixed value is specified,

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5. Heat Transfer by Air Movement

ApacheSim models the following types of air movement:

- Pre-specified air exchanges, classified as infiltration, natural ventilation or mechanical ventilation. These air exchanges may be sourced from outside air, outside air modified by a temperature offset, air at a (possibly varying) temperature defined by an *absolute profile* or air from another room. The rate of air flow is specified before the simulation, but may be made to vary with time by means of a profile. If the profile is a formula profile, the air flow rate may also vary with simulation variables such as room air temperature.

- Air flows calculated by MacroFlo. MacroFlo calculates natural ventilation air flows arising from wind and stack pressure (buoyancy). It also takes account of flow imbalances generated by HVAC systems. MacroFlo runs in tandem with ApacheSim and the calculations of the two programs are interdependent.

- Air flows specified or calculated by ApacheHVAC. Like MacroFlo, ApacheHVAC is fully integrated with ApacheSim and its ducted mechanical ventilation rates are superimposed on other air flows dealt with by ApacheSim.

The rate of heat transfer associated with a stream of air entering a space is

$$Q = mc_p(T_i - T_a) \tag{22}$$

where

m is the air mass flow rate (kg/s),

c_p is the specific heat capacity of air at constant pressure (J/kg/K),

T_i is the supply temperature of the air (ºC), and

T_a is the room mean air temperature of the air (ºC).

Equation 22 embodies an assumption that the air displaced by the supply air is at the room mean air temperature, which is consistent with ApacheSim's *stirred tank* model of the room air.

Air supply rates may be specified in various ways. A rate specified in terms of a volume flow in l/s is converted to a mass flow rate using a reference air density of 1.2 kg/m^3. A rate specified in terms of l/s/m^2 is multiplied by room floor area and then converted as above. A rate specified in terms of air changes per hour is converted to a mass flow rate m using

$$m = \rho_{ref} aV / 3600 \tag{23}$$

where

ρ_{ref} = 1.2 kg/m^3 is the reference air density,

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V is the room volume (m^3) and

a is the air change rate (ach).

The water vapour gain associated with the air supply, which plays a part in the room's latent balance, is

$$w = m(g_i - g) \tag{24}$$

where

w is the water vapour gain (kg/s)

g_i is the humidity ratio of the supply air (kg/kg)

g is the humidity ratio of the room air (kg/kg)

Carbon dioxide gain is treated in a similar way:

$$c = m(k_i - k) \tag{25}$$

where

c is the carbon dioxide gain (kg/s)

k_i is the carbon dioxide concentration of the supply air (kg/kg)

k is the carbon dioxide concentration of the room air (kg/kg)

In the case of an air supply for which the supply temperature is defined by an absolute profile, the supply humidity ratio is assumed to be that of the outside air, unless this would imply a percentage saturation of more than 90% when the supply humidity is adjusted to this limit.

The carbon dioxide concentration of outside air is assumed to take the fixed value of 360 ppm (volumetric).

The calculation of air flow rates by MacroFlo and ApacheHVAC is dealt with in the sections devoted to these programs.

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6. Long-wave Radiation Heat Transfer

6.1. Thermal Radiation Fundamentals

Building surfaces emit thermal radiation by virtue of their absolute temperature. For small surface element (dA) of a *Lambertian* emitter the radiation flux emitted into a small solid angle ($d\omega$) lying in a direction making an angle θ to the surface normal is

$$dW = \frac{1}{\pi}\varepsilon\sigma\Theta^4 \cos\theta d\omega dA$$

(26)

where

dW	is the radiation flux (W/m^2)
ε	is the surface emissivity (W/m^2)
σ	is the Stefan-Boltzmann constant (= 5.6697x10^{-8} W/m^{2}K^4)
Θ	is the absolute temperature of the surface (K)
θ	is the direction angle measured from the surface normal
$d\omega$	is an element of solid angle
dA	is an element of surface area (m^2)

Integrated over solid angle, the total radiation (W) emitted by a plane surface of area A is

$$W = \varepsilon A \sigma \Theta^4$$

(27)

Surfaces also absorb a proportion of the radiation they intercept. By Kirchhoff's law the fraction of incident radiation that is absorbed by a surface is equal to its emissivity, ε.

These results represent an idealisation of the physics of radiation emission and absorption in that they assume *Lambertian* angular characteristics and do not enter into the detail of wavelength dependence (the *grey body* assumption). However, they provide a sound basis for modelling radiation exchange in buildings.

The emission and absorption of thermal radiation by building surfaces represents an important mechanism for heat transfer. The following discussion centres on the exchange of radiation between solid surfaces. Gases and suspended particles in the air also participate in radiant exchanges and this can be important both inside and outside the building.

Thermal radiation is described as *long-wave* if it is characteristic of temperatures normally experienced in the human environment. Solar radiation lies in a shorter wavelength band and is treated separately. Surface properties are often significantly different in the long-wave and solar wavelength bands, giving rise to differences between surface emissivity and solar absorptance. Transmission properties are also strongly wavelength dependent: glass is mainly transparent to the solar spectrum but almost opaque in the long wave.

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6.2. Interior Long-wave Radiation

Radiation heat transfer between a pair of surfaces may be modelled by integrating Equation 26 over emitting area and receiving solid angle. This integration results in a *shape factor F_{1-2}* characterising radiant exchange between the surfaces:

F_{1-2} is the fraction of radiation emitted by surface 1 that reaches surface 2

By a reciprocity theorem,

$$A_1 F_{1-2} = A_2 F_{2-1}$$

(28)

where A1 and A2 are the areas of surface 1 and surface 2. Each side of this equation represents a *radiant exchange area* which can be visualised as the portion of surface 1 that effectively radiates all its energy to surface 2 (and vice versa).

By calculating shape factors and accounting for scattering (*radiosity*), it is possible to construct an accurate model of radiant heat exchange in an enclosure. For practical purposes, however, simpler models are adequate.

Models based on the concept of mean radiant temperature reduce the computational effort involved in radiant exchange calculations by a large factor. Such models introduce a single (fictitious) *radiant node* which serves as a clearing house for all surface radiant exchange transfers. In an *n*-surface enclosure this reduces the number of heat transfer pathways from approximately $\frac{1}{2} n^2$ to *n*. A variety of mean radiant temperature models have been proposed [6, 7]. In all such models the net radiant exchange between a surface and the rest of the enclosure is modelled with an equation of the form

$$W = h_r (T_s - T_{MRT})$$

(29)

where

W is the net radiative loss from the surface

h_r is a surface heat transfer coefficient for exchange with the MRT node

T_s is the surface temperature

T_{MRT} is the Mean Radiant Temperature of the enclosure

The various mean radiant temperature models differ by small amounts in the values assigned to this coefficient. ApacheSim adopts the CIBSE mean radiant temperature model [1], which provides a good representation of radiation exchange where it can be assumed that the emissivities of the surfaces bounding the enclosure do not differ greatly from one another (which is almost always the case).

In the case of holes (openings) between spaces, the holes are treated as highly conducting surfaces with solar transmittance 1 and emissivity 1. The temperature of such a surface will equilibrate at a value near the mean radiant temperature of its surroundings. This temperature will then, in turn, form part of the mean radiant temperature calculations for the adjacent spaces. This approximation has the effect of placing a diffusing medium in the opening.

It will be noted that Equation 29 is a linear function of T_s and T_{MRT}. This linearisation of the fourth-power term in Equation 29 forms part of the mean radiant temperature methodology.

6.3. The Participation of Air in Interior Radiation Exchanges

Gases present in the air, notably water vapour and to a lesser extent carbon dioxide, participate significantly in radiant exchanges within rooms. These gases radiate to their surroundings by virtue of their absolute temperature and also absorb radiation crossing the space. For these reasons it is meaningful to refer to air emissivity.

The emissivity of the air is mainly due to an infra-red absorption band associated with water vapour. A smaller contribution comes from carbon dioxide absorption.

The water vapour effect increases with humidity and room size, being a function of the product of water vapour pressure and mean path length. For a large room (for example an atrium), the effective air emissivity may be as high as 0.3. For typical rooms such as offices it is usually about 0.1. These values are highly significant. For example the figure for the atrium means that of the long-wave radiation crossing the space, 30% may be absorbed by the air.

The contribution to air emissivity from carbon dioxide is of the order 0.02 for typical CO2 concentrations and can usually be ignored for the purposes of interior radiation exchange calculations.

The most significant effect of air emissivity is its influence on radiant temperature. A radiation-absorbing air mass partially shields warm room surfaces, reducing their effect on the radiant temperature perceived by occupants.

The dependence of air emissivity on humidity also introduces a coupling effect between latent and sensible heat transfers. As the humidity rises, room surfaces exchange more heat with the air and less with each other, and heat sources behave as if their radiant fraction were reduced.

Apache Simulation models the effect of air emissivity due to water vapour. The contribution of carbon dioxide is ignored.

A model of air radiant exchanges was developed by Hottel [10], based on consideration of radiation between a surface and an adjacent hemispherical mass of gas with radius L. For gas masses of other shapes an equivalent mean beam length, L_e can be defined. For an enclosure of volume V and surface area A, L_e is approximated by

$$L_e = 3.6V / A \tag{30}$$

Hottel's model expresses the emissivity of the gas as a function of the product $p_w L_e$, where p_w is the partial vapour pressure of the participating gas. For water vapour, a good approximation to the data at standard atmospheric pressure is provided by the curve fit

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$$\ln(\varepsilon_{air}) = -0.619 - (2.958 - 0.2184\ln(p_w L_e))^2 \qquad (31)$$

Air emissivity calculated from equation 31 is used to modify the calculation of the following effects:

Inter-surface radiant exchange

Radiant exchange between surfaces and air

Distribution of radiant plant and casual gains to surfaces and air

Perceived mean radiant temperature

Solar radiation is unaffected by air emissivity, which is effectively transparent to the solar spectrum.

6.4. Exterior Long-wave Radiation

Exterior building surfaces receive long-wave radiation from the sky, the ground and other objects in the environment. They also emit thermal radiation. The difference between radiation emitted and radiation absorbed constitutes the net long-wave gain (which in most instances is negative).

The model adopted by ApacheSim for the treatment of exterior long-wave radiation follows work undertaken for the CEC European Solar Radiation Atlas [5], and endorsed by CIBSE in Guide A [1].

The net long-wave gain for an external surface of inclination β (º) is

$$L^*(\beta) = \varepsilon_e \left[L_{sky}(\beta) + L_g(\beta) - \sigma\Theta_e^4 \right] \qquad (32)$$

where

$L^*(\beta)$ is the net long-wave radiation gain (W/m²)

ε_e is the emissivity of the external surface

$L_{sky}(\beta)$ is the long-wave radiation received directly from the sky (W/m²)

$L_g(\beta)$ is the long-wave radiation received from the ground (W/m²)

Θ_e is the absolute temperature of the external surface (K)

For a horizontal surface, the long-wave radiation received from the sky is estimated from the temperature and water vapour content of the air, with a modification for cloud cover:

$$L_{sky}(0) = \sigma\Theta_a^4 \left\{ 0.904 - (0.304 - 0.061 p_w^{1/2})(1-c) - 0.005 p_w^{1/2} \right\} \qquad (33)$$

where

Θ_a is the external absolute air temperature (K)

p_w is the external air water vapour pressure (hP)

c is cloud cover (0-1)

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For an inclined surface, the long-wave radiation received directly from the sky is obtained using Cole's correlation [8]:

$$L_{sky}(\beta) = L_{sky}(0)F_{sky} + 0.09k_3(\beta)\{1 - c[0.7067 + 0.00822T_a]\}\sigma\Theta_a^4 \qquad (34)$$

where

$$F_{sky} = \cos^2(\beta/2)$$

is the shape factor from the surface to the sky

T_a is the external air temperature (ºC)

and

$$k_3(\beta) = 0.7629(.01\beta')^4 - 2.2215(.01\beta')^3 + 1.7483(.01\beta')^2 + 0.054(.01\beta') \qquad (35)$$

where

$$\beta' = \beta \qquad (\beta \leq 90)$$
$$\beta' = 180 - \beta \qquad (\beta > 90)$$

The substitution of β' for β in the expression for k3 (in contrast to the version of the formula appearing in CIBSE Guide A) avoids unphysical behaviour for slopes greater than 90° (the most important instance of which is exposed floors).

The long-wave radiation received from the ground is estimated from

$$L_g(\beta) = \sigma\{0.980\Theta_a + 0.037(1 - \rho_g)I_{glob}\}^4 F_{gnd} \qquad (36)$$

where

ρ_g is the (short-wave) ground reflectance (albedo)

I_{hglob} is the total solar flux (W/m^2) on the horizontal plane

$$F_{gnd} = 1 - F_{sky} = \sin^2(\beta/2)$$

is the shape factor from the surface to the ground

The term involving I_{hglob} makes allowance for the heating effect of solar radiation on the ground surface temperature.

Where there is diffuse shading from remote objects (as calculated by SunCast) or local shading devices (as specified in APcdb), the long-wave calculations are modified by adjusting the view factors to the sky and the ground using the diffuse sky shading factor:

$$F_{sky} = f_{shd}\cos^2(\beta/2)$$

$$F_{gnd} = 1 - F_{sky}$$

where

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f_{shd} is the diffuse sky shading factor for the surface

The assumption is that the shading objects have a radiosity equal to that of the ground.

7. Solar Radiation

7.1. Solar Radiation Fundamentals

To a good approximation, the sun is a black body radiator with a surface temperature of 5800K. The energy it radiates produces a radiation flux at the top of the earth's atmosphere which over the course of a year averages to 1353 W/m^2. Filtering by gases in the atmosphere and by cloud and particulates means that fluxes at the earth's surface are variable and typically considerably less than this figure. Further factors influencing solar radiation at ground level are varying sun angles and diffusing of the radiation by the atmosphere.

Solar radiation incident on building surfaces can be broken down into three main components: direct (beam) radiation emanating from the region of the sky near to the sun's disc, diffuse radiation from the sky vault, and radiation scattered by the ground. Direct radiation is significantly modified by shading by nearby buildings and landscape features.

Solar radiation enters the building through glazing and is absorbed (after repeated scattering) by internal surfaces. Part of this radiation may be lost by being re-transmitted out of the building through glazing. The effect of absorption and scattering by exterior surfaces (both opaque and transparent) is also significant.

7.2. Meteorological Solar Variables

ApacheSim is driven by actual weather recorded at hourly intervals and stored on a simulation weather file. The variables on the file relating to solar radiation are:

- Direct solar radiation measured perpendicular to the beam (W/m^2)
- Diffuse solar radiation measured on the horizontal plane (W/m^2)
- Solar altitude and azimuth (º)

The solar altitude and azimuth are calculated from the location of the site where the weather was recorded, together with time zone and summertime clock adjustment information. This information is also used by the programs SunCast and SunCast Lite to generate shading data for ApacheSim, and it is important that the same location data is used in both cases.

7.3. Calculation of Incident Solar Flux

ApacheSim calculates, at each time-step, the solar flux incident on every external building surface. The components of the incident flux are calculated as follows.

The direct solar flux, *Idir*, is

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$$I_{dir} = I_{beam} \cos(\theta) \qquad\qquad (\cos(\theta) > 0) \qquad\qquad (37)$$

where

I_{dir} is the direct solar flux (W/m²) incident on the surface

I_{beam} is the solar flux (W/m²) measured perpendicular to the beam

θ is the angle of incidence

The diffuse solar flux has components radiated from the sky and the ground:

$$I_{sdiff} = I_{hdiff} \cos^2(\beta/2) \qquad\qquad (38)$$

$$I_{gdiff} = \rho_g I_{hglob} \sin^2(\beta/2) \qquad\qquad (39)$$

where

I_{sdiff} is the diffuse sky solar flux (W/m²) incident on the surface,

I_{hdiff} is the diffuse sky solar flux (W/m²) on the horizontal plane,

β is the inclination of the surface,

I_{gdiff} is the diffuse ground solar flux (W/m²) incident on the surface,

ρ_g is the solar reflectance (albedo) of the ground,

$I_{hglob} = I_{hdiff} + I_{beam} \sin\alpha$ is the total solar flux (W/m²) on the horizontal plane,

α is the solar altitude.

This analysis covers the case where the sky diffuse radiation is assumed to be isotropic, the factors involving β arising from integration of this isotropic radiation over solid angle.

If the user selects the anisotropic diffuse solar radiation model from the Simulation Options menu the calculation designates a portion of the diffuse radiation circumsolar, which it treats as if it emanated from the sun position. The proportion of the diffuse radiation designated *circumsolar* varies with the intensity of the beam radiation, following a variant of the method proposed by Hay[11] with adjustments applied for diffuse shading.

7.4. *Shading & Solar Tracking*

Shading of the beam component of solar radiation may be modelled in three ways in ApacheSim:

- Shading and solar tracking calculations performed by SunCast
- Shading calculations performed by SunCast Lite
- Shading calculations performed by ApacheSim for construction-based shading devices

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SunCast and Suncast Lite shading applies to both glazed and opaque surfaces. Construction-based shading only applies to glazing.

7.5. The SunCast Shading File

Shading data generated by SunCast for the 15th day of selected months is stored on a shading file with extension '.shd'. The data for a given month comprises hourly data describing the exposure of both exterior and interior building surfaces to beam solar radiation. If the shading file is specified at run time, ApacheSim reads the data and uses it to modify the beam component of solar radiation, and when the beam enters the building through glazing, to assign it to interior surfaces. If requested in SunCast, the shading file will also contain diffuse shading factors indicating, for each exposed surface of the building, the degree of shading from the sky vault.

The data on the SunCast shading file records a shading factor for each exterior building surface receiving beam solar radiation. In the case of glazed elements, the file also records which interior surfaces are irradiated by the beam after it has passed through the glazing, and to what extent (expressed in terms of sun-patch area projected perpendicular to the beam). If a receiving surface is itself glazed, the radiation is traced on through this element to other receiving surfaces beyond, and so on. This process is referred to as *solar tracking*.

Any holes in building elements are treated by SunCast as perfectly transparent to the solar beam.

7.6. The SunCast Lite Shading File

SunCast Lite generates monthly shading data in a similar format to SunCast, but the data on its shading file is limited to external shading. It does not include solar tracking data. If a SunCast Lite shading file is specified at run time, ApacheSim uses this data to modify the beam radiation falling on exposed surfaces, but then distributes it internally as if it were diffuse radiation.

7.7. Construction-based Shading Devices

In APcdb (the constructions database manager) local shading devices may be defined for glazed constructions. These take the form of side-fins, overhangs and balconies, and may also be used to represent window recesses. Local shading devices apply to rectangular windows only, and are idealised as objects of infinite extent. An overhang, for example, is modelled as extending indefinitely to the right and left of the window.

APcdb also allows for the addition of external shutters or louvres and internal blinds or curtains. These are assigned parameters indicating their solar

characteristics and may be raised and lowered at set times or in response to variables such as solar intensity.

Construction-based shades are attached to all instances of the glazing construction. These objects shade both direct and diffuse solar radiation. They also shade long-wave sky radiation. Unlike the shading calculations performed by SunCast and SunCast Lite, the calculations for these shading devices are carried out by ApacheSim at run-time. The results of these calculations are then combined with any SunCast or SunCast Lite shading.

7.8. Distribution of Tracked Beam Solar Radiation

When a SunCast shading file is in use, and contains shading data for the current month, ApacheSim applies the shading data in the following way.

At each time-step, the radiation intercepted by each exterior receiving surface is calculated from the incident beam solar flux, taking account of the surface geometry and any external shading factor.

If the receiving surface is transparent, ApacheSim calculates the transmission and absorption of the beam. The attenuated transmitted beam is then tracked through successive interactions with building surfaces, following the tree-like data structure recorded on the shading file. Any radiation falling on an opaque element is partially absorbed and partially reflected, using an assumed solar absorptance of 0.55. Beam radiation falling on a transparent element is transmitted, absorbed and reflected in accordance with the element's properties. Radiation reflected from opaque or transparent surfaces is returned to the adjacent room for later distribution as diffuse radiation. Transmitted beam radiation is tracked on further receiving surfaces. The process terminates when all components of the beam have either encountered opaque surfaces or left the building through transparent elements. (Having left the building the beam may subsequently strike another building surface; however, ApacheSim does not currently account for such components.).

The distribution of tracked radiation is further complicated by the following factors:

1. The tracking process is actually performed twice at each time-step, to allow interpolation between the data recorded on the shading file for successive hours.

2. Adjustments are made to the projected sun-patch surface areas read from the shading file, to allow for the difference in solar position between the day being simulated and the day for which the shading calculations were performed.

3. The shading described by the factors on the shading file may be supplemented by local shading from side-fins, overhangs and balconies, as well as shading by external shutters and internal blinds.

4. The effects of window frames are accounted in the calculation of glazing transmission, absorption and reflection. Window frames use the solar absorptance value assigned in APcdb.

5. Absorption in transparent elements is split into two components, representing the effective absorption on the internal and external surfaces of the element. These components of absorption are later distributed in appropriate proportions to the adjacent spaces.

7.9. Calculation of Incident Diffuse Solar Radiation

Diffuse radiation incident on an exposed surface is the sum of components from the sky, the ground, and certain types of shading object. Shading objects block diffuse sky solar radiation to a degree determined by a *diffuse shading factor*.

Diffuse shading factors for remote shading objects are calculated optionally by SunCast (or assumed to be 1 if not calculated). This type of shading applies to both glazed and opaque surfaces.

Diffuse shading factors for construction-based shades defined in APcdb and classified as 'local' (side-fins, overhangs and balconies) are calculated for each instance of the construction occurring in the model. Where both remote and local shades apply to the same surface, their diffuse shading factors are combined by taking the lower of the two factors. This gives a conservative estimate of the degree of shading.

SunCast and 'local' shading objects are assumed to scatter ambient radiation, as well as blocking diffuse radiation from the sky. This gives rise to an additional term in the diffuse incident flux. For the purpose of estimating this flux, shading objects are assumed to be vertically oriented, adjacent to a large vertical wall, and both wall and shading object are assumed to have a reflectance of 0.3. Ground reflection is accounted for, but direct and circumsolar radiation is excluded from the calculation.

Construction-based shades of the 'external' type (shutters and louvres) have a sky shading factor and a ground shading factor set in APcdb (both of which may optionally be calculated from the direct shading characteristics). These factors attenuate the radiation incident on the glazed element from the sky, the ground and the other types of shading object. Radiation scattered by shading devices of this type is ignored.

7.10. Distribution of Diffuse Solar Radiation

The diffuse component of solar radiation incident on an external glazed element – the sum of components from the sky and the ground – is partially transmitted and partially absorbed in the element. The transmitted portion is distributed over the interior building surfaces as follows.

In simple cases the diffuse radiation entering a room through a glazed element is distributed over the other surfaces in the room in proportion to their areas. An exception to this rule may apply in the case of glazed, external receiving surfaces. If the shape factor implied by the area-weighted distribution is greater than the maximum theoretical shape factor between the receiving surface and the source surface (given their areas and relative orientation) the shape factor is reduced to the theoretical maximum. The radiation deficit is then spread over the other receiving surfaces in proportion to their estimated shape factors. This exception prevents windows in the same façade from radiating directly to each other. Such windows are treated effectively as one large window.

Surfaces receiving diffuse radiation distributed in this way reflect, absorb and (if transparent) transmit it in appropriate proportions. 'Holes' are treated as perfectly transparent. Reflected diffuse radiation is combined with any reflected tracked radiation in the room and distributed over the room's surfaces on the basis of an *acceptance weighting* in which each surface is irradiated in proportion to its area multiplied by the sum of its absorptance and its transmittance. The resulting distribution emulates the distribution that would result from apportioning the radiation on a strictly area-weighted basis and successively repeating the process for the reflected components until no radiation remained.

Radiation transmitted through transparent partitions in the course of these processes is treated in a similar way to radiation entering the building from outside. No shape factor adjustment is applied, however.

A portion of any radiation distributed to external windows is lost by transmission back out of the building.

The above steps are repeated up to 10 times to distribute the diffuse radiation through the building. Any residual radiation at the end of the process is assigned to room surfaces in a final modified *acceptance* distribution.

7.11. Special Element Adjacencies

Windows in elements assigned the adjacencies 'Outside air with offset temp.' and 'Temp. from profile', and those in partition elements linking to inactive spaces, receive no external solar radiation. Any radiation transmitted out of the building through such windows in the course of the solar distribution is lost from the model.

7.12. Distribution of Non-tracked Beam Solar Radiation in Apache

When solar tracking does not apply (that is, when a SunCast shading file is not in use, or is in use but contains no data for the current month) beam radiation is distributed over the building interior in the same way as diffuse radiation.

7.13. Solar Transmission, Absorption & Reflection by Glazing

7.13.1. Glazing Solar Characteristics

The characteristics of glazed constructions are calculated from first principles from data entered in APcdb.

The data for a glazed construction is as follows:

- Glazing data:

  ε_e External surface emissivity (used if not set for external pane)

  ε_i Internal surface emissivity (used if not set for internal pane)

  R_e Optional user-specified external surface resistance (m^2K/W)

  R_i Optional user-specified internal surface resistance (m^2K/W)

- Layer data:

  d_j Thickness of glazing layer j

  λ_j Conductivity of glazing layer j (W/mK)

  Gas$_j$ Gas filling cavity (used with layer thickness and orientation to calculate convection coefficient when the latter is not specified explicitly)

  h_{cj} Convection coefficient for cavity (W/m^2K) (used with adjacent pane emissivities to calculate cavity resistance when the latter is not specified explicitly)

  r_j Thermal resistance of cavity (m^2K/W) (may be user-specified or derived from cavity resistance and adjacent pane emissivities)

  τ_j Normal-incidence solar transmittance of glazing layer j

  ρ_{ej} Normal-incidence solar reflectance of glazing layer j for radiation incident from outside

  ρ_{ij} Normal-incidence solar reflectance of glazing layer j for radiation incident from inside

  n_j Refractive index of glazing layer j

  ε_{ej} Outside surface emissivity of glazing layer j

  ε_{ij} Inside surface emissivity of glazing layer j

- Frame data:

  f Frame area as percentage of total area (%)

  R_f Frame resistance, excluding surface resistances (m^2K/W)

  α_f Frame solar absorptance

- Local, external and internal shading data

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From this data the program calculates the following derived parameters for the construction as a whole:

- Solar transmission, absorptance and reflectance parameters at 10 angles of incidence spaced at 10 degree intervals.
- Parameters describing the distribution of solar absorption within the construction
- Separate U-values for the glazing and the frame, and a net U-value combining the glazing and frame U-values.

These calculations, which are carried out in a pre-simulation stage, are performed as follows.

For each glazing layer (pane) the solar characteristics are checked for consistency. An analysis based on the Fresnel equations is carried out for a pane having the given layer refractive index and an absorption parameter (extinction coefficient) that is adjusted to match the given pane absorptance. This is done for two rays with perpendicular polarisations, and the results are combined to give normal-incidence transmittance, absorptance and reflectance values. These are then compared with the values entered for these parameters. The most likely cause of a discrepancy in this comparison is the presence of a reflecting film on the glass surface. In this case the discrepancy is corrected by adding a reflecting film with properties chosen to match the characteristics entered. When the discrepancy cannot be corrected by a modification of this sort, the refractive index is adjusted to produce a match. The derived characteristics are then used to produce transmittance, absorptance and reflectance parameters for 10 incidence angles, again using the Fresnel equations applied to rays of two polarisations.

The solar characteristics of the construction as a whole are then calculated for the 10 incidence angles and the two polarised rays. This process in general involves consideration of an infinite number of reflections at glazing/air interfaces. The result is a set of solar transmission, absorptance and reflectance parameters at 10 angles of incidence, the absorptance characteristics being further resolved according to where in the construction the absorption occurs. The absorption parameters are then simplified, without any compromise of accuracy, by replacing each absorption by equivalent absorptions at the external internal surface of the constructions, using an equivalent circuit representation involving the thermal resistances of the layers.

7.13.2. Glazing Solar Interactions during Simulation

During a simulation, whenever solar radiation strikes a glazed surface the interaction of the radiation with the glazing is calculated using the construction's solar parameters.

Portions of the incident radiation are transmitted, absorbed, and reflected. For direct (beam) radiation the appropriate angular characteristics are used. For diffuse radiation, the calculation uses hemispherically averaged characteristics obtained by integrating over the incidence angles with a weighting for solid angle. Any frame forming part of the construction is assumed to have a transmittance of zero and an absorptance as set in the frame data.

External shutters/louvres and internal blinds/curtains participate in the interaction according to their parameters as specified in APcdb.

7.14. Solar Absorption & Reflection by Opaque Surfaces

External opaque building surfaces absorb and reflect solar radiation according to their solar absorptance as assigned in APcdb. SunCast and SunCast Lite shading data is applied to external opaque surfaces, and SunCast shading data is applied also to internal opaque surfaces.

8. Casual Gains

Casual gains to rooms are specified in the Thermal View of the Virtual Environment. They are classified for convenience according to type:

- Fluorescent lighting
- Tungsten lighting
- Machinery
- Miscellaneous
- Cooking
- Computers
- People

Gains have in general both sensible and latent components. They may be expressed in terms of either absolute values or on a floor area basis. Each component of the gain is specified as a maximum value modulated by a percentage profile.

Sensible gains add sensible heat to the room (or remove it, if negative). A portion of this heat, dictated by the *radiant fraction*, is in the form of thermal radiation and the remainder – the convective portion – is input to the room air. The radiant portion is allocated to the room surfaces on an area-weighted basis.

Latent gains add water vapour to the room air. All gains are expressed in terms of an energy input, the conversion factor between latent gain and water vapour addition being the latent heat of vaporisation of water.

9. Thermo-physical Properties of Air

9.1. Psychrometric Conversions

The modelling of psychrometric processes in ApacheSim, MacroFlo and ApacheHVAC requires a set of calculation procedures for converting between such quantities as:

- Dry-bulb temperature
- Wet-bulb temperature
- Dew-point temperature
- Relative humidity
- Percentage saturation
- Humidity ratio
- Water vapour pressure
- Enthalpy
- Air density
- Atmospheric pressure

For these conversions ApacheSim, MacroFlo and ApacheHVAC use published procedures set out in Reference 9.

9.2. Air Moisture Content Dynamics

The storage of water vapour in the room air mass is represented by

$$w_a = \rho_a dg / dt \tag{40}$$

where

ρ_a is the air density (kg/m^3)

w_a is the net water vapour gain to the air (kg/s)

g is the room air humidity ratio (kg/kg)

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10. Room Plant & Control

10.1. Models of Room Control

The simulation of heating, cooling and humidity control systems may be achieved in ApacheSim by two alternative means:

- Idealised room control
- Detailed system simulation using ApacheHVAC

The capabilities available within ApacheHVAC are described in a separate section. Here we deal with the idealised temperature and humidity control that can be modelled by ApacheSim operating in standalone mode.

10.2. Idealised Control of Room Temperature

The idealised plant model provided within ApacheSim is illustrated in Figure 1, which shows a graph of sensible heat (Q_{sens}) supplied to a room as a function of room temperature (T_{room}). The shape of the plant characteristic is defined by four parameters:

T_h Simulation Heating Set-point (ºC)

T_c Simulation Cooling Set-point (ºC)

$Q_{h\,max}$ Simulation Heating Capacity (kW)
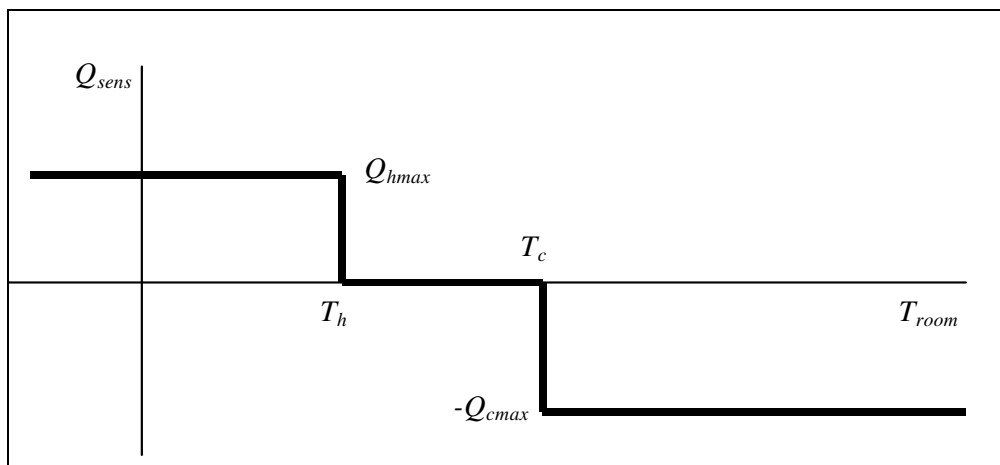
$Q_{c\,max}$ Simulation Cooling Capacity (kW)

Figure 1: Idealised room temperature control

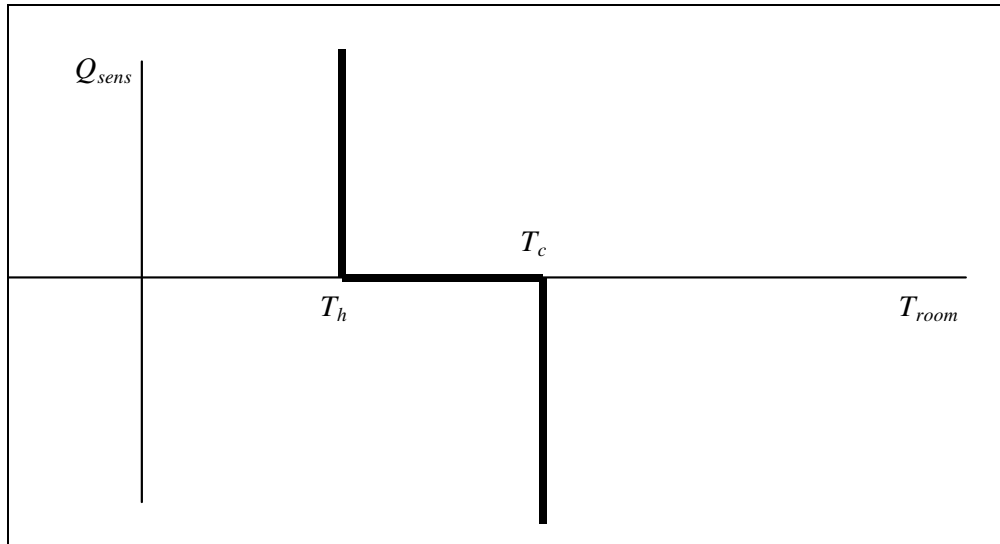**\<Virtual Environment\> design, simulate + innovate**

Figure 2 Idealised room temperature control without capacity limits

With no limit on heating or cooling capacity (Q_{hmax} and Q_{cmax} effectively infinite), the characteristic is as shown in Figure 2. In this case the plant has the effect of constraining to be in one of the following three states:

$T_{room} = T_h$, with heating being supplied as required, or

$T_{room} = T_c$, with cooling being supplied as required, or

$T_h < Tr_{oom} < T_c$ (in the dead-band), with no heating or cooling supplied.

When a limit is imposed on either Q_{hmax} or Q_{cmax} (Figure 1), two further possibilities exist:

$T_{room} < T_h$, with heating at maximum, or

$T_{room} > T_c$, with cooling at maximum.

To provide additional flexibility, the Simulation Heating and Cooling Set-points may vary with time. This is done using absolute profiles. If this option is chosen it is important to ensure that the condition $T_h \leq T_c$ is satisfied at all times.

The model is further elaborated by the additional parameters:

- Control Temperature Radiant Fraction
- Heating Device Radiant Fraction
- Cooling Device Radiant Fraction
- Heating Profile
- Cooling Profile

Control Temperature Radiant Fraction specifies the characteristic of the room thermostat, a value of 0 (the most common setting) denoting an air temperature sensor. The radiant fractions for heating and cooling specify the

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fraction of the output of the heating and cooling devices that is in the form of thermal radiation. Heating and cooling profiles allow the periods of plant operation to be specified.

10.3. Idealised Control of Room Humidity

An idealised model of room humidity control is provided by the parameters:

- Minimum Percentage Saturation
- Maximum Percentage Saturation

During periods of cooling plant operation (as dictated by the Cooling Profile), the room percentage saturation will be held between these limits by the addition or removal of water vapour. Setting Minimum Percentage Saturation to 0% disables humidification control and setting Maximum Percentage Saturation to 100% disables dehumidification control.

11. Room & Building Heat Balance

ApacheSim uses a *stirred tank* model of the air in a room. This means that the calculations are based on the concepts of bulk air temperature and humidity, which are assumed to be uniform within the room. If a finer spatial resolution of these variables is required, it can be achieved by subdividing the room, or through the use of computational fluid dynamics (for example MicroFlo).

The task of determining thermal conditions throughout the building proceeds by balancing sensible and latent heat flows entering and leaving each air mass and each building surface. If ApacheHVAC is included in the simulation, the solution process also incorporates thermal balances for each of the system components. If either ApacheHVAC or MacroFlo is included, it also includes the mechanical and natural ventilation air flow rates calculated by these programs, and the inter-dependence between these variables and those calculated within ApacheSim.

11.1. Sensible Heat Balance

The balancing of heat flows for the air in each room involves the following components:

- Thermal storage in the air and the furniture (Section 3).
- Convection from the room surfaces (Section 4).
- Heat transfer by air movement (of the three types identified in Section 0).
- The convective portion of casual gains (Section 8).
- The convective portion of any plant input – idealised (Section 9) or from ApacheHVAC.

By equating the sum of these components to zero, a heat balance is established at the room *air node*.

Further heat balances are set up for each of the interior room surfaces. The components of the surface heat flow balance are:

- Heat conduction out of the building element (Section 3).
- Convection to the surface from the room air (Section 4).
- Thermal radiation exchanged with the radiant temperature node (Section 6).
- Solar gain absorbed by the surface (Section 7).
- The surface's share of the radiant portion of casual gains (Section 8).
- The surface's share of radiant plant input – idealised (Section 9) or from ApacheHVAC.

The use of a mean radiant temperature model of long-wave radiant heat exchange means that a further heat balance is also required at the radiant temperature node, equating all heat flows there to zero.

Finally, there is a heat balance for every exterior surface of the building, involving:

- Heat conduction out of the building element (Section 3).
- Convection to the surface from the outside air (Section 4).
- Thermal radiation exchanged with the external environment (Section 6).
- Solar gain absorbed by the surface (Section 7).

The heat balance equations are solved using linear algebra techniques. Because some of the equations are nonlinear, iteration is used to converge on a global solution.

11.2. Latent Heat Balance

The equations dealing with the modelling of air humidity are relatively simple.

A balance of water vapour flows is established for the air in each room, involving:

- Water vapour transfer by air movement (of the three types identified in Section 5).
- The latent portion of casual gains (Section 8).
- The dynamics of water vapour storage in the air (Section 9).
- Any plant humidification or dehumidification – idealised (Section 10) or from ApacheHVAC.

11.3. Carbon Dioxide Balance

The modelling of carbon dioxide concentration follows the same pattern as latent heat balance.

A balance of carbon dioxide flows is established for the air in each room, involving:

- Carbon dioxide transfer by air movement (of the three types identified in Section 5).
- The carbon dioxide input associated with People casual gains.
- The dynamics of carbon dioxide storage in the air.

Carbon dioxide transport by air exchange, MacroFlo and ApacheHVAC air flows is modelled by

$$c = m(k_i - k)$$

(41)

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where

c is the carbon dioxide gain (kg/s)

k_i is the carbon dioxide concentration of the supply air (kg/kg)

k is the carbon dioxide concentration of the room air (kg/kg)

Carbon dioxide inputs to rooms are linked to People casual gains. The sum of the sensible and latent heat inputs from people is taken as an indicator of metabolic output, and linked to carbon dioxide generation in the following way.

The metabolic output associated with different levels of physical activity is measured in MET units. A seated person at rest has an output of 1 MET. This corresponds to a heat output (sensible plus latent) of 58.2 Watts per square metre of body surface. As the metabolic rate increases, heat output rises in proportion. Assuming a body surface area of 1.8 m^2 for an average adult, the metabolic heat output is therefore

Q_{met} = 58.2 x 1.8 M = 104.76 M

where

Q_{met} is metabolic heat output (Watt)

M is metabolic rate (MET).

Carbon dioxide output is also assumed to rise in proportion to metabolic rate, and to take a value of 0.005 l/s (0.3 l/minute) for a metabolic rate of 1.2 MET (a figure corresponding to typical office work):

N = 0.005 M /1.2

where

N = CO2 output (l/s)

Combining these two relations gives CO_2 output as a function of total sensible and latent heat output:

N = 0.005 (Q_{met} /104.76) /1.2 = 3.977x10^{-5} Q_{met}

Q_{met} is obtained from the People casual gains data.

The dynamics of carbon dioxide storage in the air are treated in a similar way to the dynamics of moisture storage (see Sections 5 and 9):

$$c_a = \rho_a dk / dt$$
(42)

where

ρ_a is the air density (kg/m^3)

c_a is the net carbon dioxide gain to the air (kg/s)

k is the room air carbon dioxide concentration (kg/kg)

For reporting purposes, k is converted to a volumetric concentration expressed in parts per million.

If, as a result of a low or zero ventilation rate, the calculated room carbon dioxide concentration rises above 200000 ppm it is capped at this value.

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The calculation of room carbon dioxide concentration is based on the assumption that the outside air has a fixed volumetric carbon dioxide concentration of 360 ppm.

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